PHYSICS STUDY GUIDE
CHAPTER 10: WORK-ENERGY

TOPICS:
- Work
- Power
- Kinetic Energy
- Gravitational Potential Energy
- Elastic Potential Energy
- Conservation of Mechanical energy

DEFINITIONS

- **WORK**: Potential to do something (A transfer of energy into or out of the system).
- **POWER**: rate at which work is done
- **KINETIC ENERGY:** Ability to do work as a result of the velocity of the system.
- **GRAVITATIONAL POTENTIAL ENERGY:** Ability to do work as result of the height between two objects (Earth and an object).
- **ELASTIC POTENTIAL ENERGY:** Ability to do work as a result of the elongation of an elastic material

WHAT YOU MUST KNOW
- Work: Direction of the force (or component of the force) must be parallel to the direction of the displacement.
- Kinetic energy: it is associated with the velocity of the object.
- Gravitational Potential Energy: it is associated with the height of the object with respect to Earth.
- Elastic Potential Energy: it is associated with the elongation of the elastic material.

PHYSICAL QUANTITIES

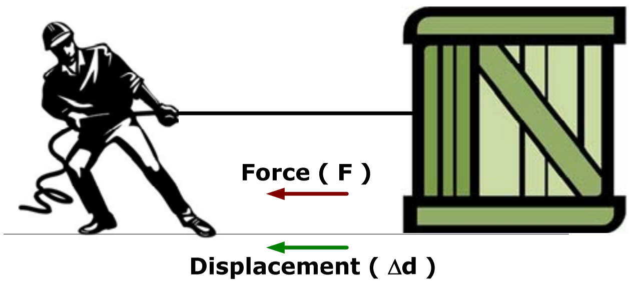
SYMBOL	NAME	UNITS
W	Work	Joules (J)
F	Force	Newton (N)
Δd	Displacement	Meters (m)
P	Power	Watt (W)
Δt	Clock Reading	Seconds (s)
KE	Kinetic Energy	Joules (J)
m	Mass	Kilograms (kg)
v	Velocity	Meters per second (m/s)
U_G	Gravitational Potential Energy	Joules (J)
g	Gravitational Constant	Meters per second squared (m/s^2)
dy	Height	Meters (m)
U_S	Elastic Potential Energy	Joules (J)
k	Spring constant	Newton per meter (N/m)
ΔL	Elongation	Meters (m)

MATHEMATICAL MODELS

- See last page

WORK

- Potential to do something such as break a piece of chalk, break someone's foot, dent a guardrail, etc.
- Work is the Force exerted over some displacement.
- Force needs to be exerted in the same direction of the displacement ($w = F \cdot \Delta d$)



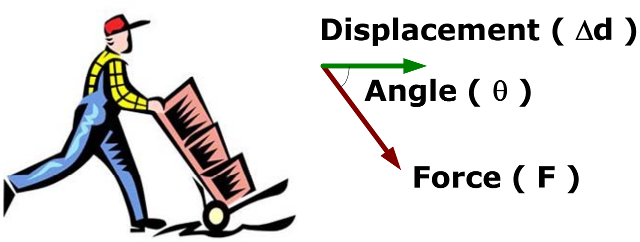
The worker exerts a force of 400 N over a displacement of 6 m.

$w = F \cdot \Delta d$

$w = 400\ N \cdot 6\ m$

$w = 24000\ J$

- When force is exerted at an angle, only the component parallel to the displacement will be used ($w = F \cdot \cos\theta \cdot \Delta d$)



The worker exerts a force of 400 N over a displacement of 6 m at an angle of 60°.

$w = F \cdot \cos\theta \cdot \Delta d$

$w = 400\ N \cdot \cos(60°) \cdot 6\ m$

$w = 400\ N \cdot 0.5 \cdot 6\ m$

$w = 12000\ J$

POWER

- Rate at which work is done

Two different lifting machines do 19600 J of work lifting a car to change a tire.

Machine A does the work in 20 s and Machine B does the work in 16 s.

$P = \frac{w}{\Delta t}$ $\qquad$ $P = \frac{w}{\Delta t}$

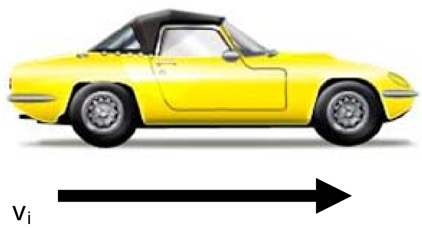
$P = \frac{19600\ J}{20\ s}$ $\qquad$ $P = \frac{19600\ J}{16\ s}$

$P = 980\ W$ $\qquad$ $P = 1225\ W$

- Two different machines do the same work (19600 J)
- Machine B does the work faster than machine A.
- Machine B develops a power of 1225 w whereas machine A develops a power of 980 w.

KINETIC ENERGY

- **KINETIC ENERGY:** Ability to do work as a result of the velocity of the system.
- Energy associated with the velocity (v) of an object.
- **Example:** A cool 1200 kg yellow car is running at 45 m/s. As a sharp turn is coming ahead the driver slows down to 20 m/s.



v_i

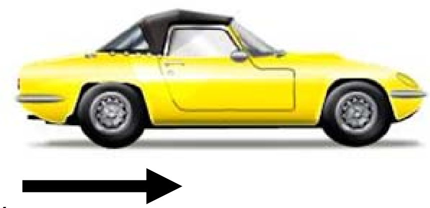
$m = 1200$ kg

$v_i = 45$ m/s

$$KE_i = \frac{m \cdot (v_i)^2}{2}$$

$$KE_i = \frac{1200 \cdot (45)^2}{2}$$

$KE_i = 1{,}215{,}000$ J



v_F

$m = 1200$ kg

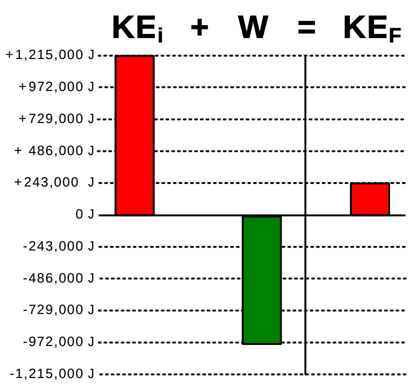
$v_F = 20$ m/

$$KE_F = \frac{m \cdot (v_F)^2}{2}$$

$$KE_F = \frac{1200 \cdot (20)^2}{2}$$

$KE_F = 240{,}000$ J

Work-energy bar chart



Work-kinetic energy theorem

$KE_i + W = KE_F$

$1{,}215{,}000 \text{ J} + W = 240{,}000 \text{ J}$

$W = -975{,}000$ J

Explanation:

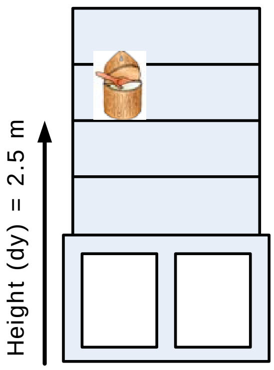
The cool car starts with a kinetic energy of 1,275,000 J and ends with a kinetic energy of 240,000 J

A negative work of 975,000 J needs to be done on the car to slow it down.

The work done on the car changes the energy of the car

GRAVITATIONAL POTENTIAL ENERGY

- **GRAVITATIONAL POTENTIAL ENERGY:** Ability to do work as result of the height between two objects (Earth and an object).
- Energy associated with the vertical distance (dy) between the object and Earth.
- **Example:** it is dinner time. Simon brings a 2 kg canister with rice from the pantry (2.5 m above the ground) to the counter (1.2 m above the ground).



$m = 2$ kg

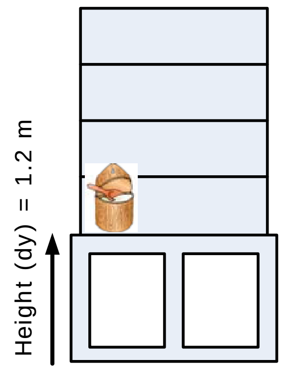
$g = 9.8\ m/s^2$

$dy_i = 2.5$ m

$U_{Gi} = g \cdot m \cdot d_{yi}$

$U_{Gi} = 9.8\ m/s^2 \cdot 2\ kg \cdot 2.5\ m$

$U_{Gi} = 49$ J



$m = 2$ kg

$g = 9.8\ m/s^2$

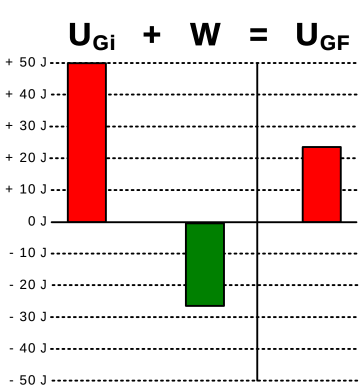
$dy_F = 1.2$ m

$U_{GF} = g \cdot m \cdot d_{yF}$

$U_{GF} = 9.8\ m/s^2 \cdot 2\ kg \cdot 1.2\ m$

$U_{GF} = 23.52$ J

Work-energy bar chart



Work-Gravitational Potential energy theorem

$U_{Gi} + W = U_{GF}$

$49\ J + W = 23.52\ J$

$W = -25.48\ J$

Explanation:

The system Earth-canister starts with a gravitational potential energy of 49 J and ends with a gravitational potential energy of 23.52 J

A negative work of 25.48 J is done bringing the canister down to the counter.

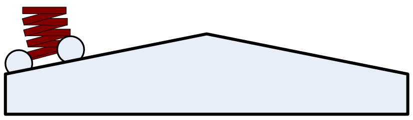
The work done on the canister changes the potential of the canister to do something.

ELASTIC POTENTIAL ENERGY

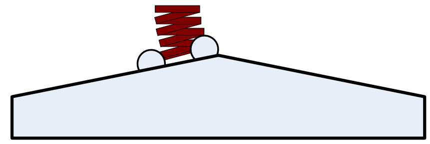
- **ELASTIC POTENTIAL ENERGY:** Ability to do work as a result of the elongation of an elastic material

- Energy associated with the elongation (ΔL) of an elastic material.

- **Example:** A conqueror of the hill project uses a spring with a spring constant of 2300 N/m to propel his PEEMO to the top of the hill. To achieve this goal, the spring must be compressed 0.06 m. The students notice that at the top of the hill the spring is still compressed 0.01 m.



Elongation (ΔL) = 0.06 m



Elongation (ΔL) = 0.01 m

$k = 2300$ N/m

$\Delta L_i = 0.06$ m

$$U_{Si} = \frac{k \cdot (\Delta L_i)^2}{2}$$

$$U_{Si} = \frac{2300 \cdot (0.06)^2}{2}$$

$U_{Si} = 4.14$ J

$k = 2300$ N/m

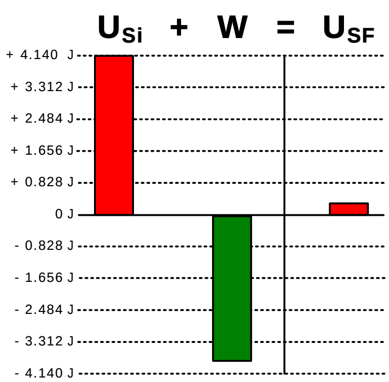
$\Delta L_F = 0.01$ m

$$U_{SF} = \frac{k \cdot (\Delta L_i)^2}{2}$$

$$U_{SF} = \frac{2300 \cdot (0.01)^2}{2}$$

$U_{SF} = 0.115$ J

Work-energy bar chart



Work-Elastic Potential energy theorem

$U_{Si} + W = U_{SF}$

$4.140 \text{ J} + W = 0.115 \text{ J}$

$W = -4.025$ J

Explanation:

The PEEMO starts with an elastic potential energy of 4.140 J and ends with an elastic potential energy of 0.115 J

A negative work of 4.025 J is done on the PEEMO.

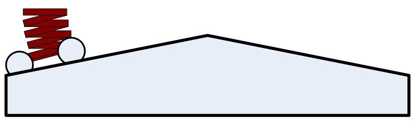
The work done on PEEMO changes the potential of the PEEMO to do something.

CONSERVATION OF MECHANICAL ENERGY

- **MECHANICAL ENERGY:** The addition of all types of mechanical energy ($TE = KE+U_G+U_S$) present in an enclosed system.

- **Example:** A conqueror of the hill project uses a spring with a spring constant of 600 N/m to propel his PEEMO to the top of the hill. To achieve this goal, the spring must be compressed 0.08 m. The students notice that at the top of the hill the spring is still compressed 0.02 m. The bottom of the hill has a height of 0.02 m and the top of the hill has a height of 0.09 m.

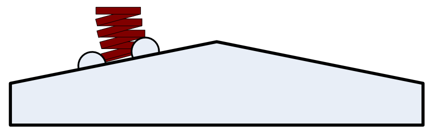
BOTTOM OF THE HILL



m = 1.5 kg
k = 600 N/m
g = 9.8 m/s^2

v = 0 m/s
dy = 0.02 m
ΔL = 0.08m

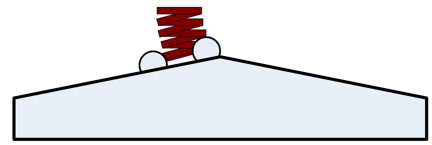
MIDWAY



m = 1.5 kg
k = 600 N/m
g = 9.8 m/s^2

v = ?
dy = 0.055 m
ΔL = 0.05 M

TOP OF THE HILL



m = 1.5 kg
k = 600 N/m
g = 9.8 m/s^2

v = ?
dy = 0.09 m
ΔL = 0.02 m

Use your knowledge to find KE, UG & US

$KE = \frac{m \cdot (v)^2}{2}$ $\quad U_G = g \cdot m \cdot d_y$ $\quad U_S = \frac{k \cdot (\Delta L)^2}{2}$ $\quad KE + U_G + U_S = TE$

- Remember TE is conserved (stays the same).

- No energy can be created, no energy can be destroyed.

	KE	**U_G**	**U_S**	**TE**	SPEED	HEIGHT	ELONGATION
BOTTOM	0	0.2940	1.92	2.214	0	0.02	0.08
MIDWAY	?	0.8085	0.75	2.214	?	0.055	0.05
TOP	?	1.3230	0.12	2.214	?	0.09	0.02

- Find the missing kinetic energies by finding the difference from the total energy.

$KE + U_G + U_S = TE$

KE + 0.8085 + 0.75 = 2.214

KE = 0.6555 J

$KE + U_G + U_S = TE$

KE + 1.3230 + 0.12 = 2.214

KE = 0.771 J

- Find the missing velocities.

$v = \sqrt{\frac{2 \cdot KE}{m}}$ $\quad v = \sqrt{\frac{2 \cdot (0.6555)}{1.5}}$ $\quad$ v=0.93 m/s

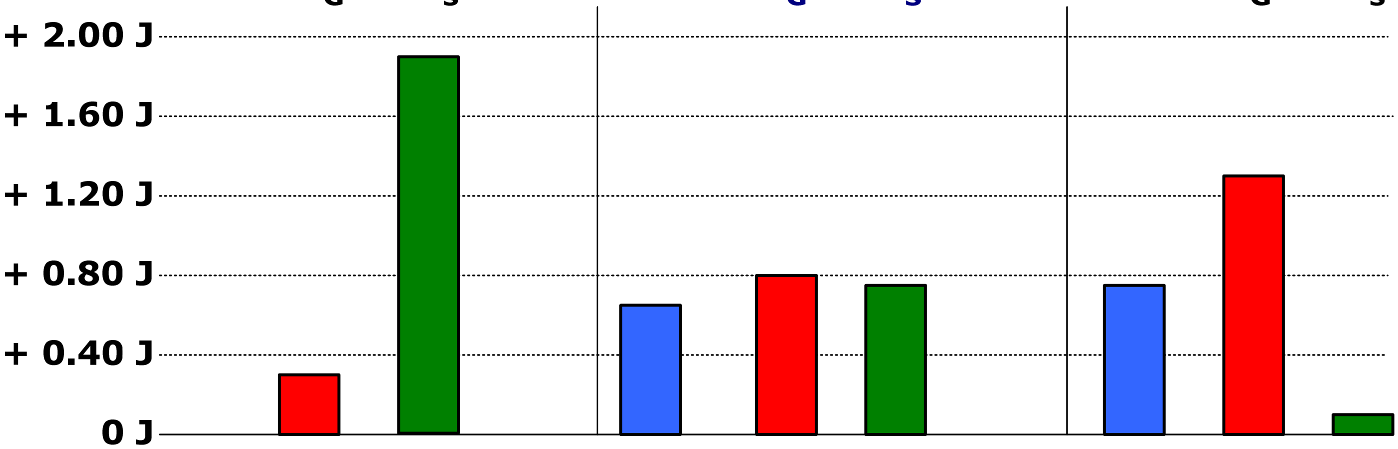
$v = \sqrt{\frac{2 \cdot KE}{m}}$ $\quad v = \sqrt{\frac{2 \cdot (0.771)}{1.5}}$ $\quad$ v=1.01 m/s

- **NOTE: add rows to the chart or create a new chart if you need to make more predictions.**

Work-energy bar chart

BOTTOM OF THE HILL **MIDWAY** **TOP OF THE HILL**

$$KE + U_G + U_s + w = KE + U_G + U_s + W = KE + U_G + U_s$$



Explanation:

- Conservation of mechanical energy:
    - At the bottom of the hill the PEEMO is at rest therefore there is no kinetic energy (blue bar), the height of the PEEMO from the hill is very low, so is the gravitational potential energy (red bar) of the system, the spring or rubber bands are elongated and a big proportion of elastic potential energy is stored (green bar).
    - Elastic potential energy is converted into gravitational potential energy as the PEEMO goes up the hill and the height of the PEEMO with respect to the ground changes.
    - Elastic potential energy is also converted into kinetic energy as the car gains some velocity going up the hill (how much elastic potential energy is converted to kinetic energy will depend on the mechanism of propulsion).
    - The total energy of the system remains constant because no more energy was created and no energy was destroyed.

MATHEMATICAL MODELS WORK - ENERGY	NAME:

Work	$w = F \cdot \Delta d$
Work (force exerted at an angle)	$w = F \cdot \cos\theta \cdot \Delta d$
Power	$P = \frac{w}{\Delta t}$
Kinetic Energy	$KE = \frac{m \cdot (v)^2}{2}$
Work Kinetic Energy Theorem	$KE_i + w = KE_F$
Gravitational Potential Energy	$U_G = g \cdot m \cdot d_y$
Work Gravitational Potential Energy Theorem	$U_{Gi} + w = U_{GF}$
Elastic Potential Energy	$U_S = \frac{k \cdot (\Delta L)^2}{2}$
Work Elastic Potential Energy Theorem	$U_{Si} + w = U_{SF}$
Total Mechanical Energy	$KE + U_G + U_S = TE$
Work Energy Principle	TEi + W = TEF $KE_i + U_{Gi} + U_{Si} + W = KE_F + U_{GF} + U_{SF}$