One Dimensional Motion

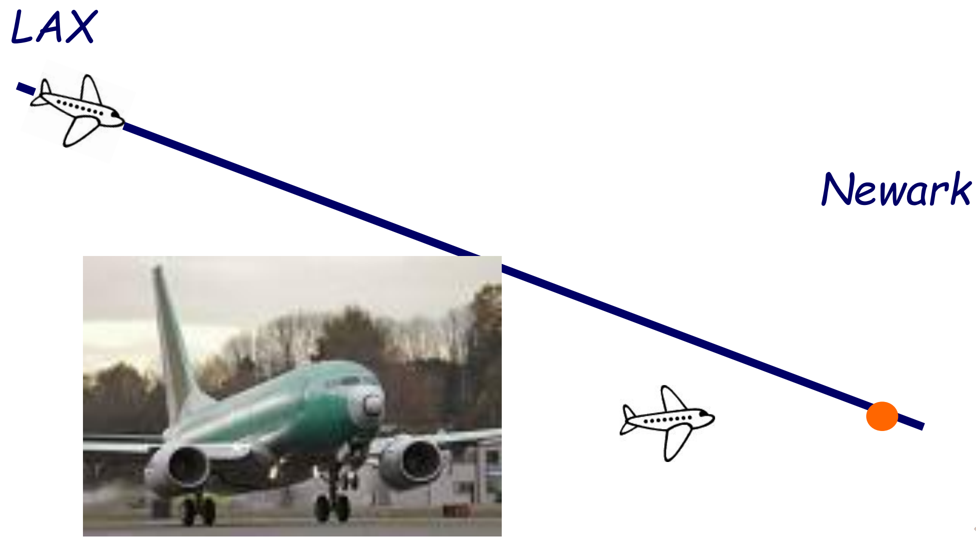
Displacement, Velocity, and Acceleration

Motion along a straight line

- Motion
- Position and displacement
- Average velocity and average speed
- Instantaneous velocity and speed
- Acceleration
- Constant acceleration: A special case
- Free fall acceleration

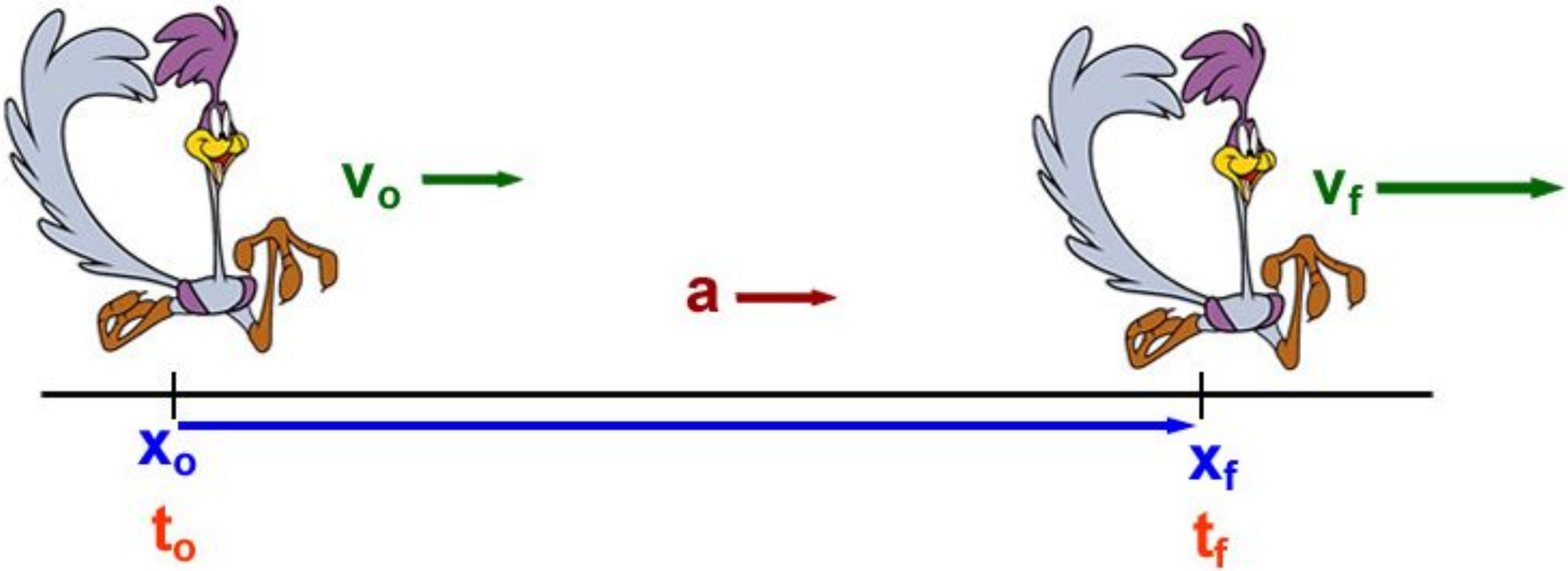
Assumptions

- We will first consider only motion along a straight line.
- Simplification: Consider a moving object as a particle, i.e. a "point object"



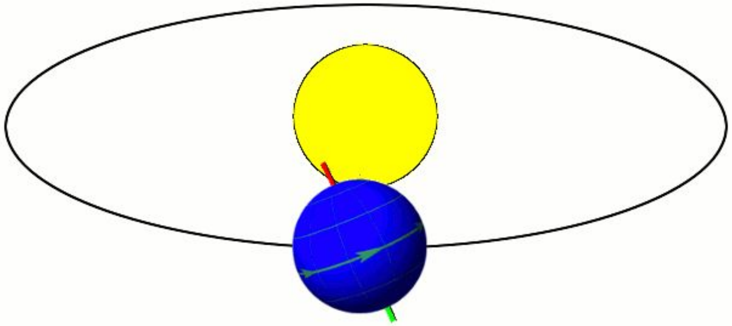
4 Basic Quantities in Kinematics

Displacement, Velocity, Time and Acceleration



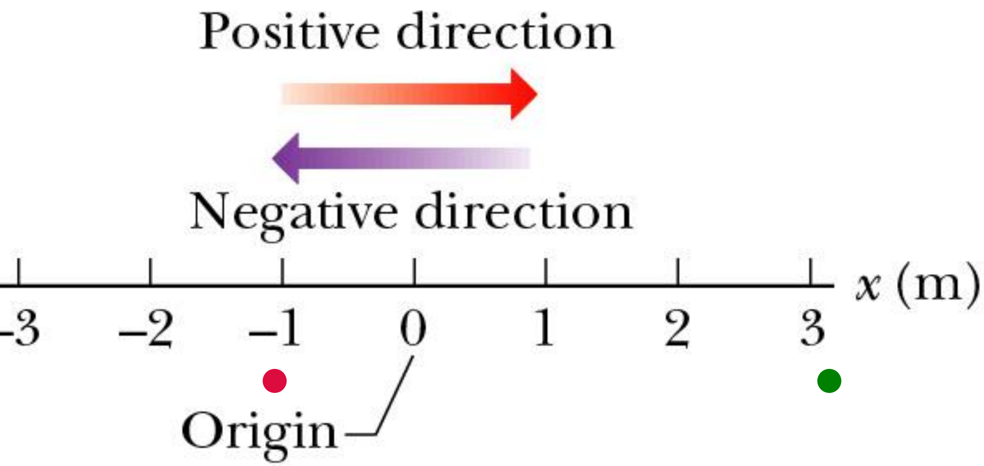
Motion

- Motion - the change of position with respect to a frame reference.
- Frame of reference – a selected system in which we measure everything from



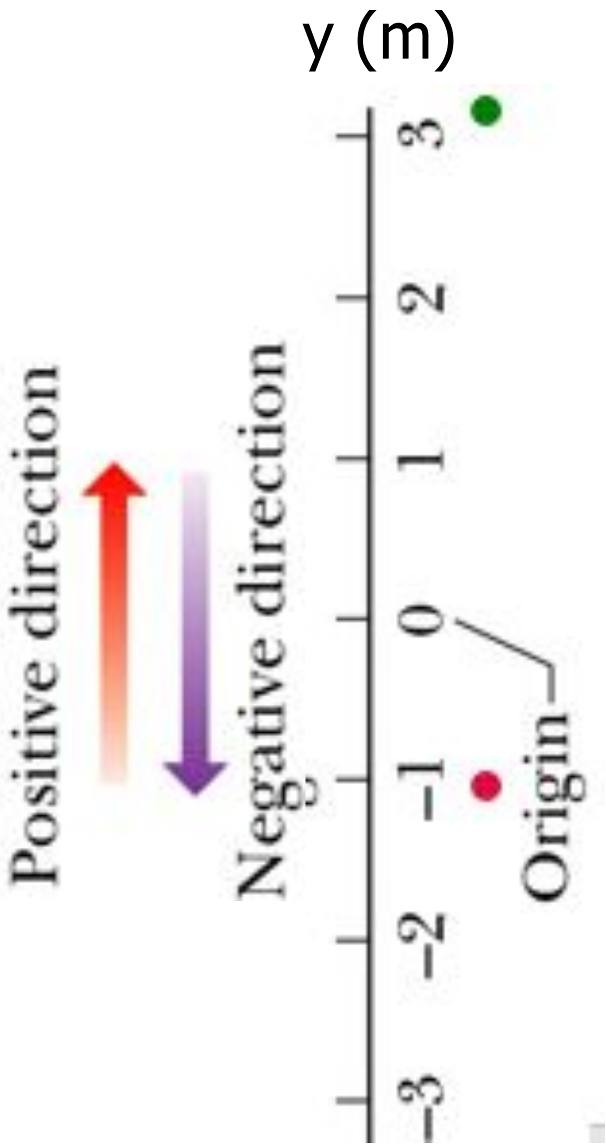
Position

- How can we represent position along a straight line?
- Position definition:
  - Defines a starting point: origin (x = 0), x relative to origin
  - Direction: positive (right or up), negative (left or down)
  - It depends on time: t = 0 (start clock), x(t=0) does not have to be zero.
- Position has units of [Length]: meters.



x = + 3.1 m

x = - 1 m

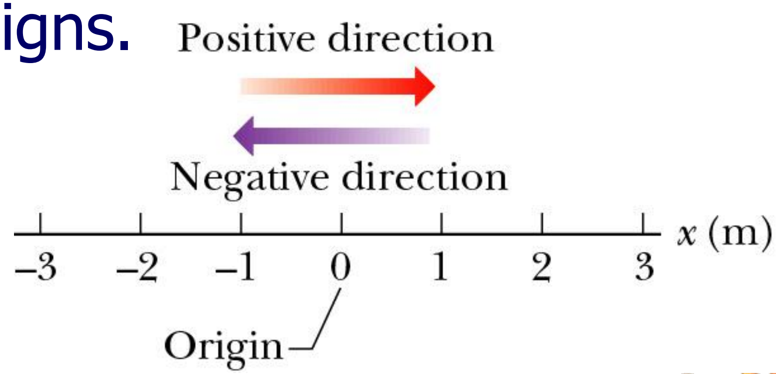


y = + 3.1 m
Or
3.1 m up

x = - 1 m
Or
1 m down

Vector and Scalar

- A vector quantity is characterized by having both a magnitude and a direction.
  - Displacement, Velocity, Acceleration, Force …
  - Denoted in boldface type $\mathbf{v}$, $\mathbf{a}$, $\mathbf{F}$ … or with an arrow over the top $\vec{v}$, $\vec{a}$, $\vec{F}$ … .
- A scalar quantity has magnitude, but no direction.
  - Distance, Mass, Temperature, Time …
- For motion along a straight line, the direction is represented simply by + and – signs.
  - + sign: Right or Up.
  - − sign: Left or Down.



Vectors and Scalars

Scalars

- Distance
- Speed
- Acceleration?

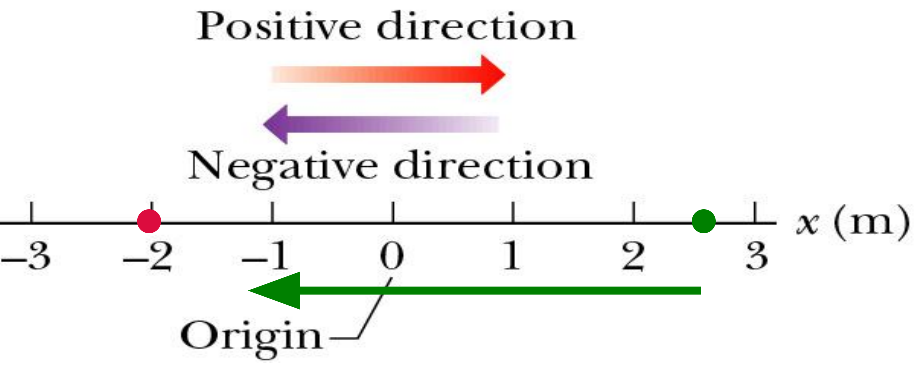
Vectors

- Displacement
- Velocity
- Acceleration

Displacement

- Displacement is a change of position in time.
- Displacement: $\Delta x = x_f - x_i$

- It is a vector quantity.
- It has both magnitude and direction: + or – sign
- It has units of [length]: meters.



$x_1 = + 2.5$ m

$x_2 = - 2.0$ m

$\Delta x = -2.0 \text{ m} - 2.5 \text{ m} = -4.5 \text{ m}$

OR

$\Delta x = 4.5$ m left

Displacement

- Aaron leaves Physics and walks 10. meters west. Then he turns and walks 30. meters east. What is Aaron's total displacement from Physics?

Distance and Position-time graph

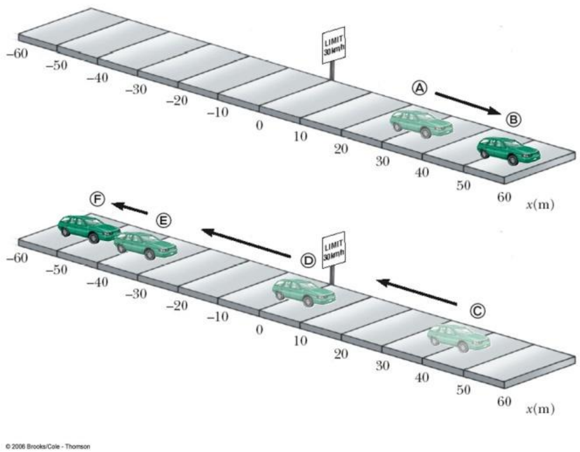
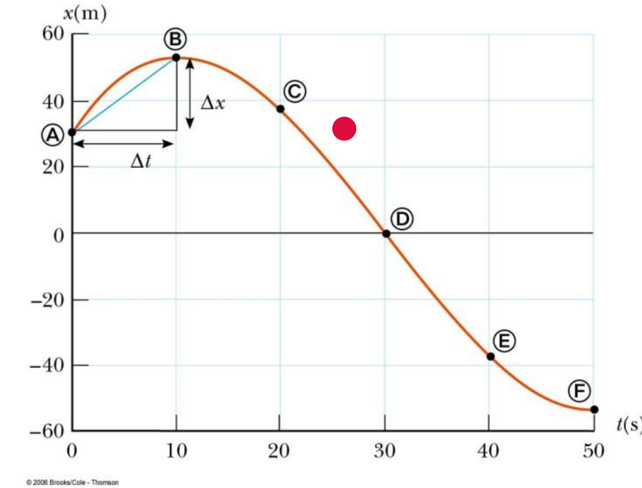


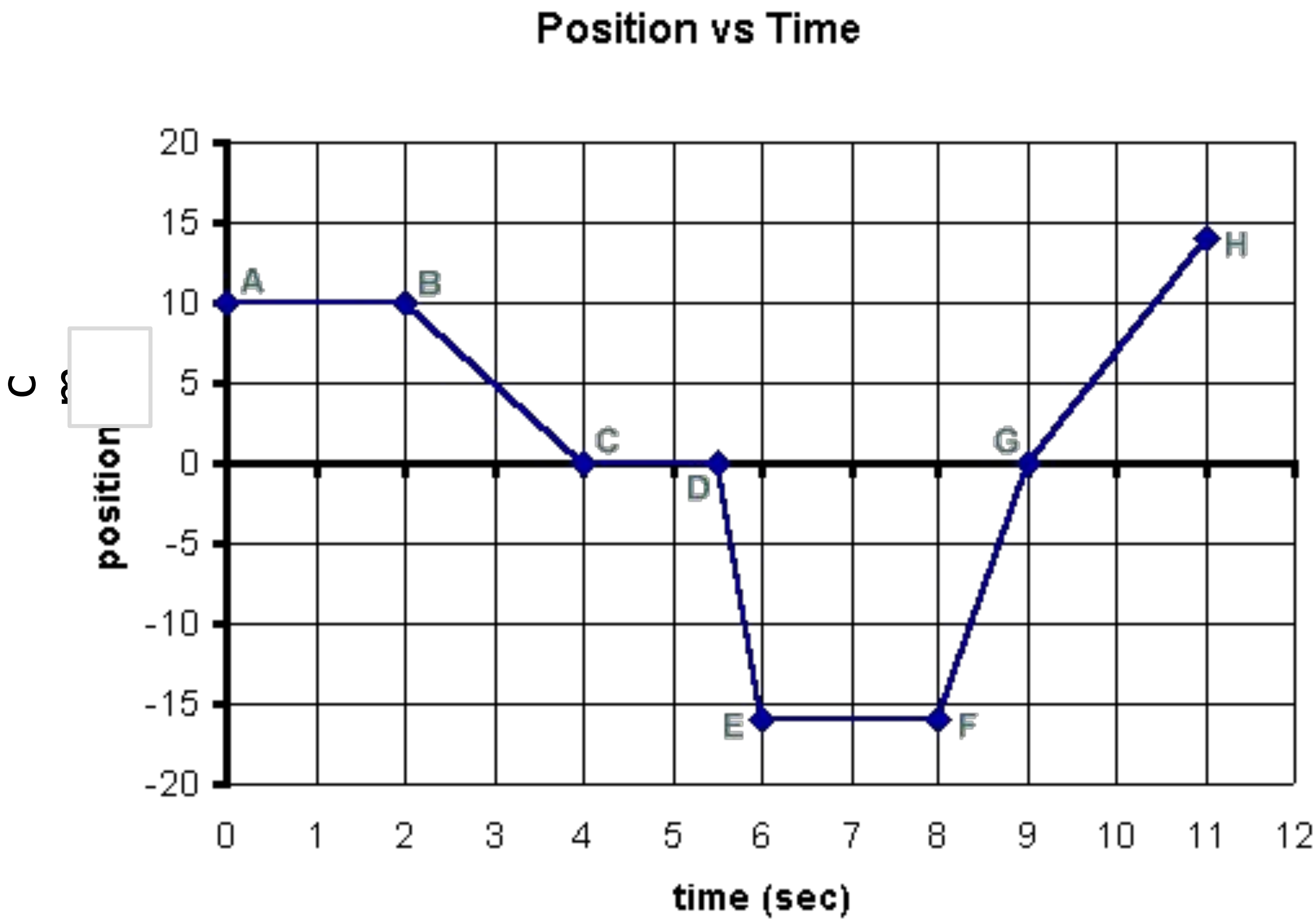
TABLE 2.1

Position of the Car at Various Times

Position	t (s)	x (m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	– 37
Ⓕ	50	– 53



- Displacement in space
  - From A to B: $\Delta x = x_B - x_A = 52\text{ m} - 30\text{ m} = 22\text{ m}$
  - From A to C: $\Delta x = x_c - x_A = 38\text{ m} - 30\text{ m} = 8\text{ m}$
- Distance is the length of a path followed by a particle
  - from A to B: $d = |x_B - x_A| = |52\text{ m} - 30\text{ m}| = 22\text{ m}$
  - from A to C: $d = |x_B - x_A| + |x_C - x_B| = 22\text{ m} + |38\text{ m} - 52\text{ m}| = 36\text{ m}$
- Displacement is not Distance.


Position vs Time
position
time (sec)
20
15
10
5
0
-5
-10
-15
-20
0
1
2
3
4
5
6
7
8
9
10
11
12
A
B
C
D
E
F
G
H

Velocity

- Velocity is the rate of change of position.
- Velocity is a vector quantity.
- Velocity has both magnitude and direction.
- Velocity has a unit of [length/time]: meter/second.

  - Average velocity

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

  - Average speed

$$s_{\text{avg}} = \frac{\text{total distance}}{\Delta t}$$

  - Instantaneous velocity

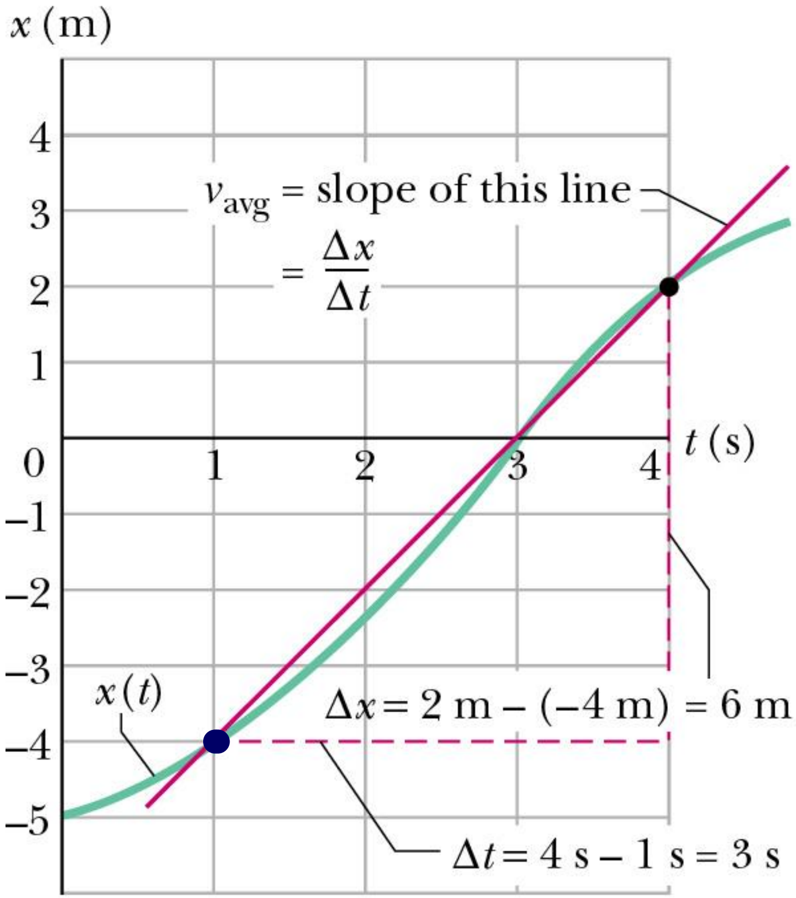
$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

displacement

distance

displacement

Average Velocity



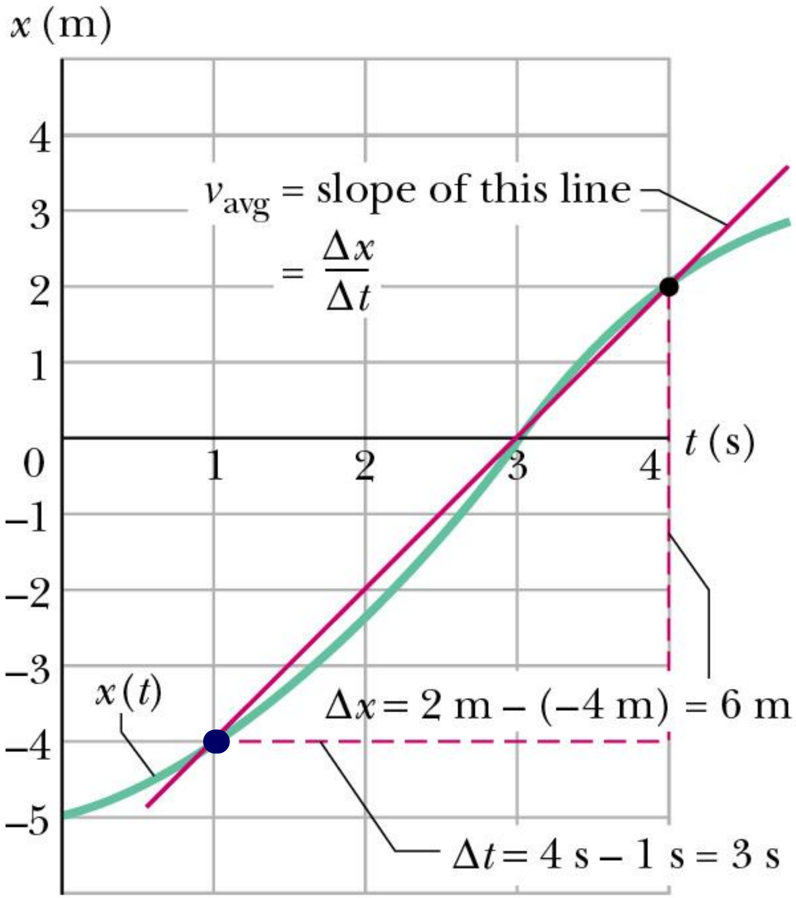
- Average velocity

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

is the slope of the line segment between end points on a graph.

- Dimensions: length/time -> [m/s].
- SI unit: m/s.
- It is a vector.

Average Speed



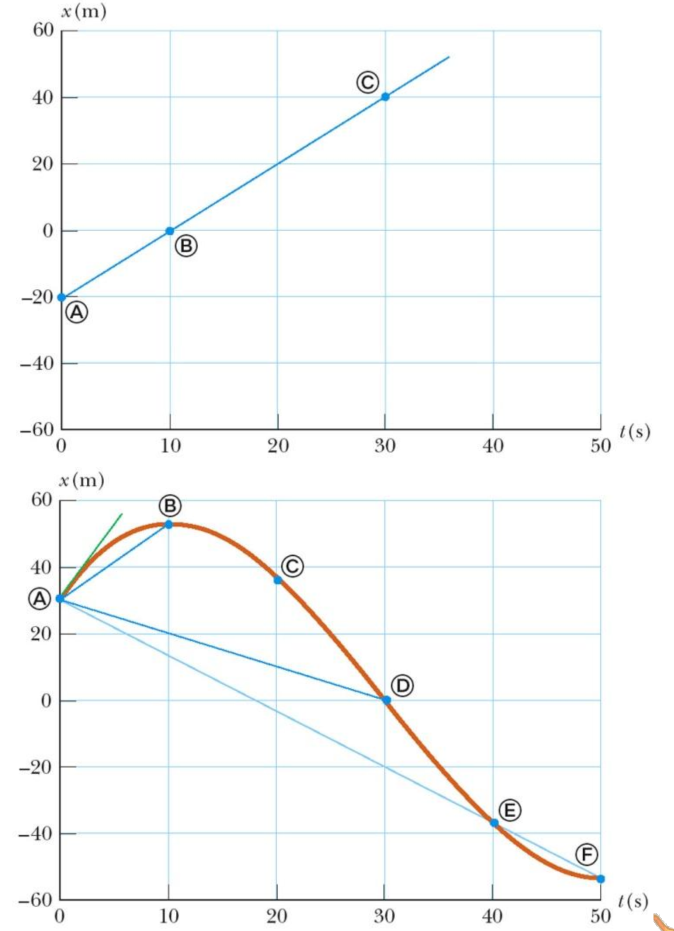
- Average speed

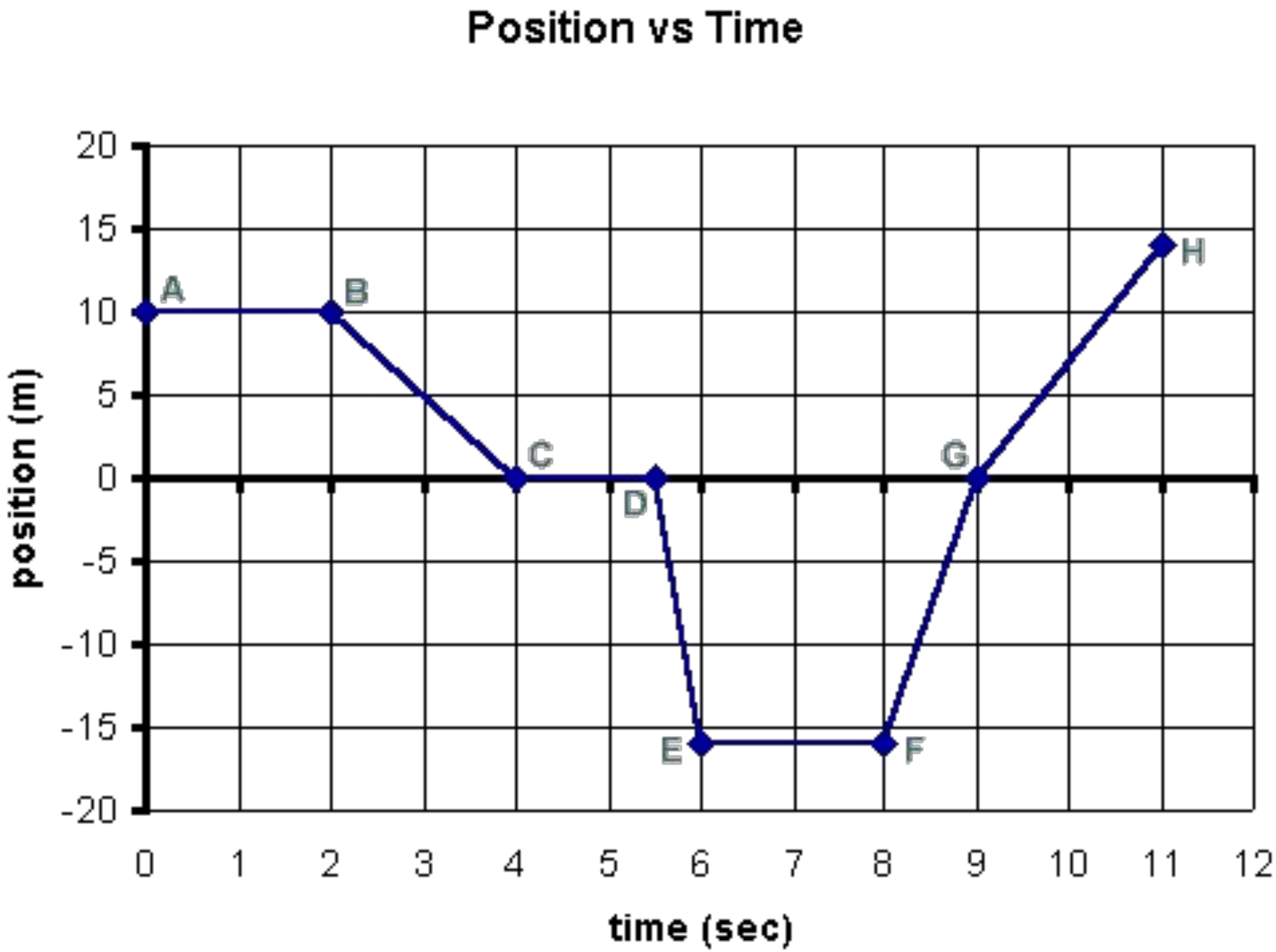
$$s_{avg} = \frac{\text{total distance}}{\Delta t}$$

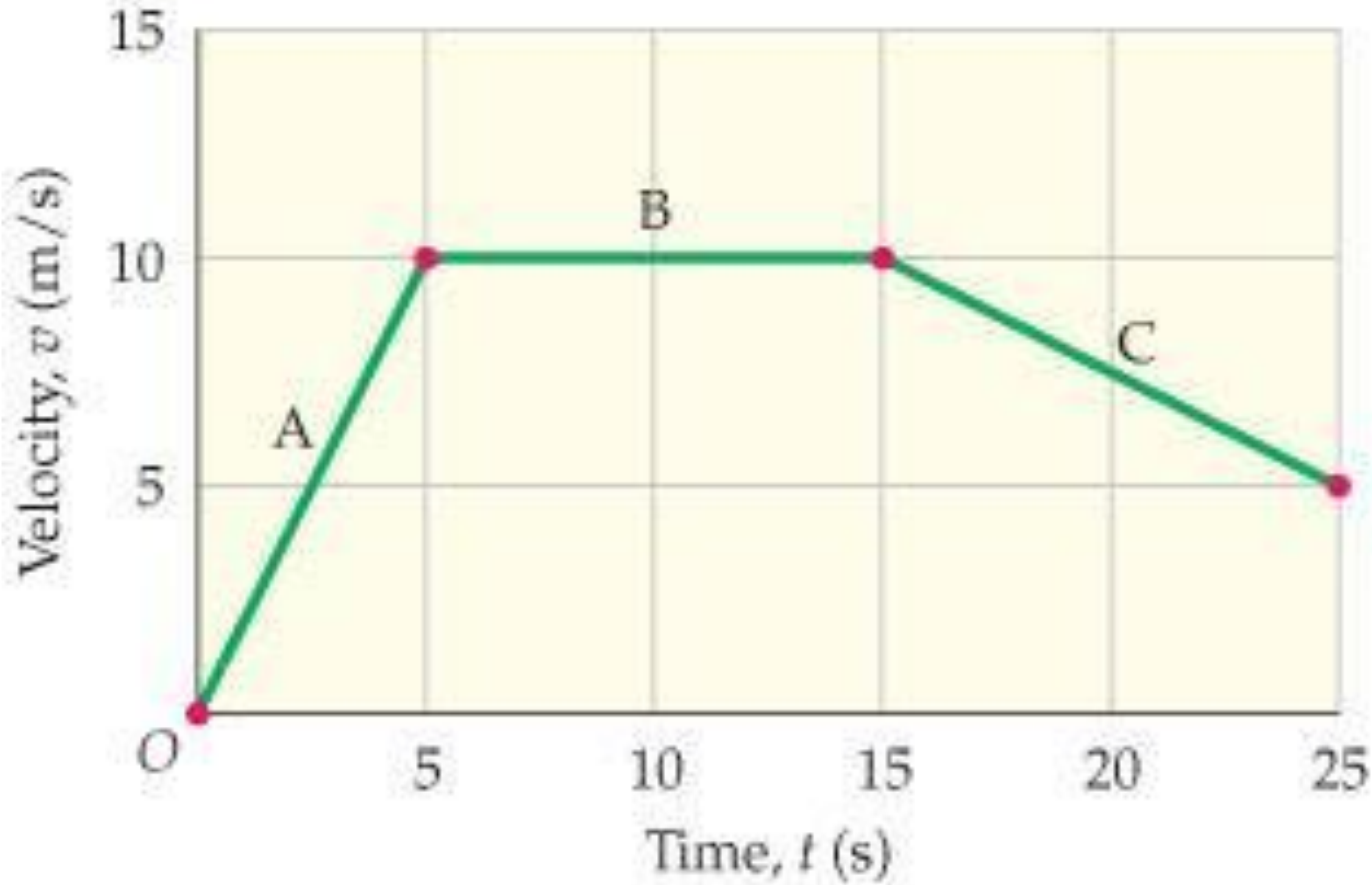
- Dimension: length/time, [m/s].
- Scalar: No direction involved.
- Not necessarily = V_{avg}:
  - S_{avg} = (6m + 6m)/(3s+3s) = 2 m/s
  - V_{avg} = (0 m)/(3s+3s) = 0 m/s

Graphical Interpretation of Velocity

- Velocity can be determined from a position-time graph
- Average velocity equals the slope of the line joining the initial and final positions. It is a vector quantity.
- An object moving with a constant velocity will have a graph that is a straight line.



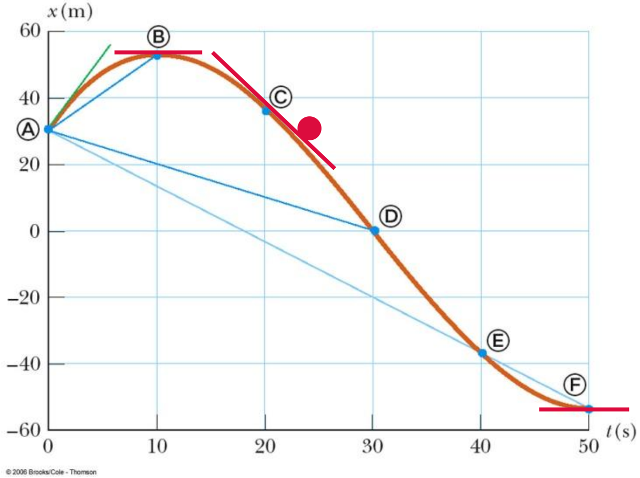

Position vs Time
position (m)
time (sec)
20
15
10
5
0
-5
-10
-15
-20
0
1
2
3
4
5
6
7
8
9
10
11
12
A
B
C
D
E
F
G
H


15
10
5
O
Velocity, v (m/s)
A
B
C
5
10
15
20
25
Time, t (s)

Instantaneous Velocity

- Instantaneous means "at some given instant". The instantaneous velocity indicates what is happening at every point of time.
- Limiting process:
  - Chords approach the tangent as Δt => 0
  - Slope measure rate of change of position
- Instantaneous velocity:
- It is a vector quantity.

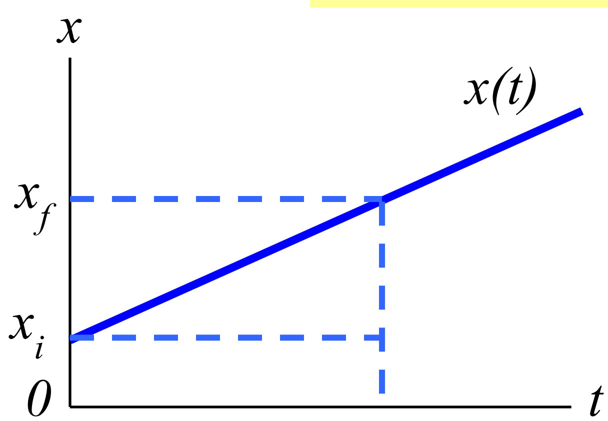
$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

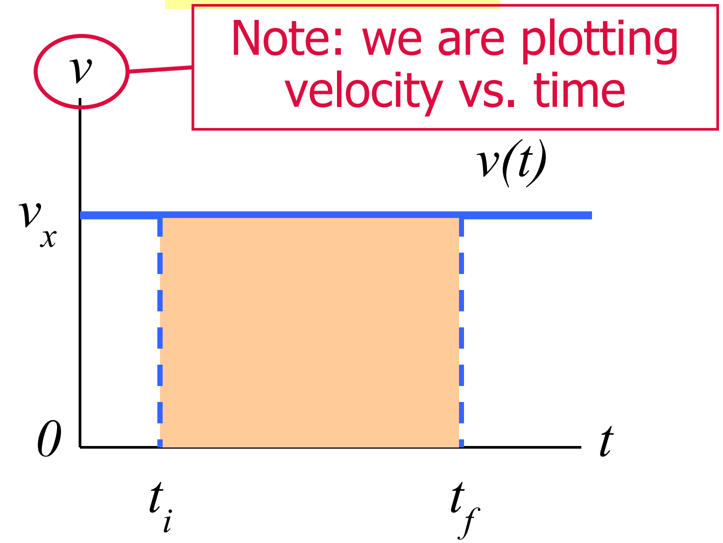


- Dimension: length/time (L/T), [m/s].
- It is the slope of the tangent line to x(t).
- Instantaneous velocity v(t) is a function of time.

Uniform Velocity

- Uniform velocity is the special case of constant velocity
- In this case, instantaneous velocities are always the same, all the instantaneous velocities will also equal the average velocity
- Begin with $v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$ then $x_f = x_i + v_x \Delta t$





Average Acceleration

- Changing velocity (non-uniform) means an acceleration is present.
- Acceleration is the rate of change of velocity.
- Acceleration is a vector.
- Acceleration has both magnitude and direction.
- Acceleration has a dimensions of length/time^2: [m/s^2].
- Definition:
  - Average acceleration $$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$
  - Instantaneous acceleration $$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\frac{dx}{dt} = \frac{d^2 v}{dt^2}$$

Average Acceleration

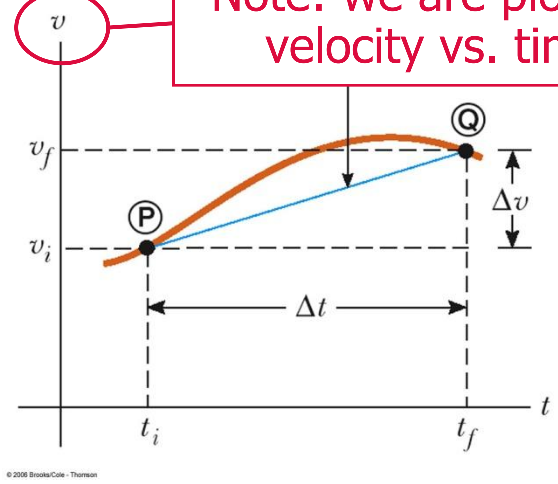
- Average acceleration

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

- Velocity as a function of time

$$v_f(t) = v_i + a_{avg}\Delta t$$

Note: we are plotting velocity vs. time



- Average acceleration is the slope of the line connecting the initial and final velocities on a velocity-time graph

Describing Motion

- It is tempting to call a negative acceleration a "deceleration," but that is not always the case.

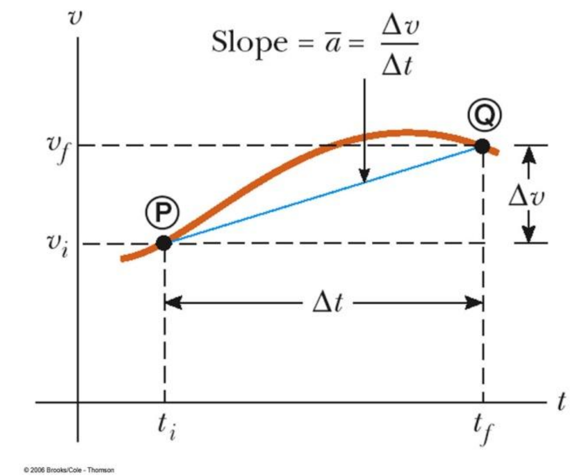
Initial velocity	Acceleration	Motion
+	+	Speeding up
-	-	Speeding up
+	-	Slowing down
-	+	Slowing down
- or +	0	Constant velocity
0	- or +	Speeding up from rest
0	0	Remaining at rest

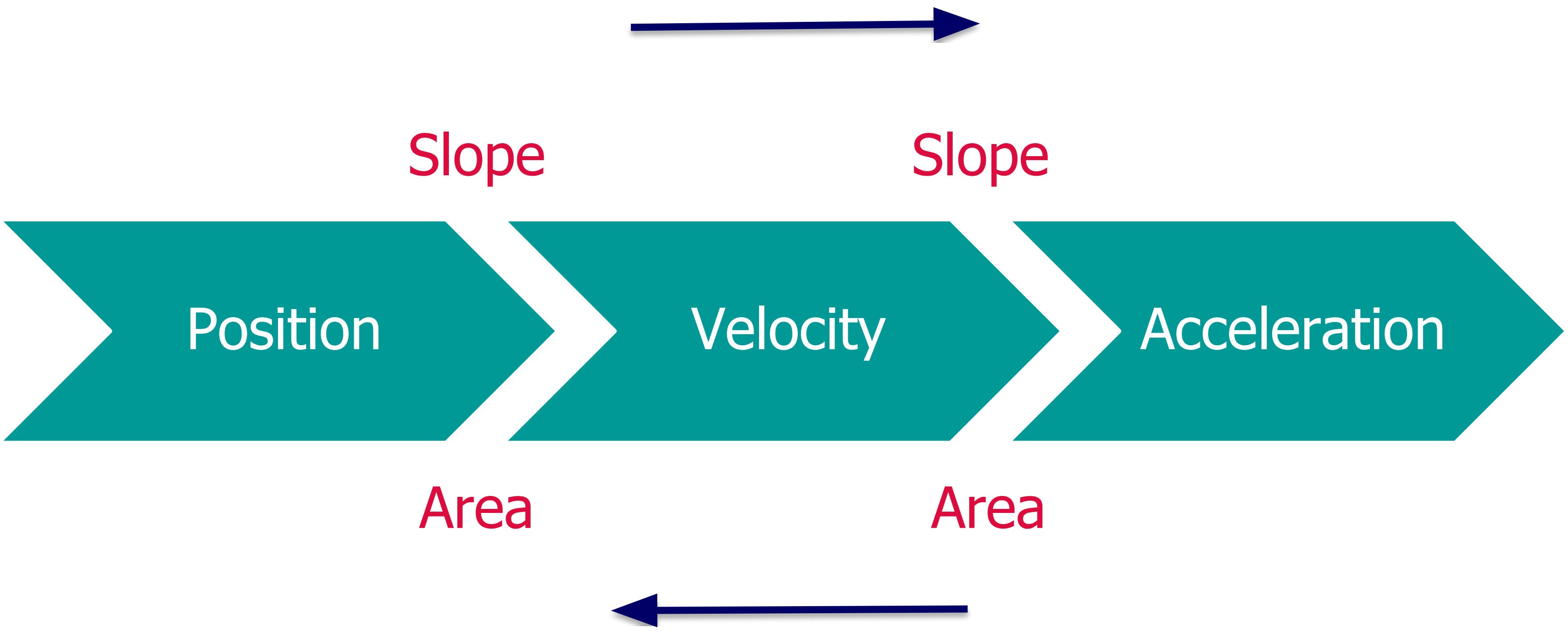
Instantaneous and Uniform Acceleration

- The limit of the average acceleration as the time interval goes to zero

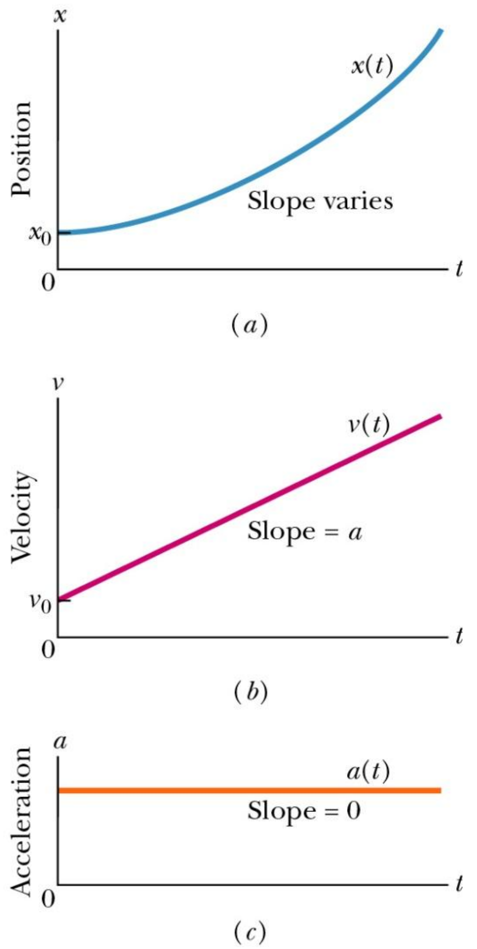
$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\frac{dx}{dt} = \frac{d^2 v}{dt^2}$$

- When the instantaneous accelerations are always the same, the acceleration will be uniform. The instantaneous acceleration will be equal to the average acceleration
- Instantaneous acceleration is the slope of the tangent to the curve of the velocity-time graph




Slope
Slope
Position
Velocity
Acceleration
Area
Area

Special Case: Motion with Uniform Acceleration (our typical case)



- Acceleration is a constant
- Kinematic Equations (which we will derive in a moment)

$$v = v_0 + at$$

$$\Delta x = \bar{v}t = \frac{1}{2}(v_0 + v)t$$

$$\Delta x = v_0 t + \tfrac{1}{2}at^2$$

$$v^2 = v_0{}^2 + 2a\Delta x$$

Derivation of the Equation (1)

- Given initial conditions:
  - $a(t) = \text{constant} = a,\ v(t=0) = v_0,\ x(t=0) = x_0$
- Start with definition of average acceleration:

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - t_0} = \frac{v - v_0}{t - 0} = \frac{v - v_0}{t} = a$$

- We immediately get the first equation

$$v = v_0 + at$$

- Shows velocity as a function of acceleration and time
- Use when you don't know and aren't asked to find the displacement

Derivation of the Equation (2)

- Given initial conditions:
  - $a(t) = \text{constant} = a,\ v(t=0) = v_0,\ x(t=0) = x_0$
- Start with definition of average velocity:

$$v_{avg} = \frac{x - x_0}{t} = \frac{\Delta x}{t}$$

- Since velocity changes at a constant rate, we have

$$\Delta x = v_{avg} t = \frac{1}{2}(v_0 + v)t$$

- Gives displacement as a function of velocity and time
- Use when you don't know and aren't asked for the acceleration

Derivation of the Equation (3)

- Given initial conditions:
  - $a(t) = \text{constant} = a$, $v(t=0) = v_0$, $x(t=0) = x_0$

- Start with the two just-derived equations:

$$v = v_0 + at \qquad \Delta x = v_{avg}t = \frac{1}{2}(v_0 + v)t$$

- We have $\Delta x = \frac{1}{2}(v_0 + v)t = \frac{1}{2}(v_0 + v_0 + at)t$ $\quad \Delta x = x - x_0 = v_0 t + \frac{1}{2}at^2$

- Gives displacement as a function of all three quantities: time, initial velocity and acceleration
- Use when you don't know and aren't asked to find the final velocity

Derivation of the Equation (4)

- Given initial conditions:
  - $a(t) = \text{constant} = a$, $v(t=0) = v_0$, $x(t=0) = x_0$
- Rearrange the definition of average acceleration

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t} = a$$

to find the time

$$t = \frac{v - v_0}{a}$$

- Use it to eliminate t in the second equation:

$$\Delta x = \frac{1}{2}(v_0 + v)t = \frac{1}{2a}(v + v_0)(v - v_0) = \frac{v^2 - v_0{}^2}{2a}$$

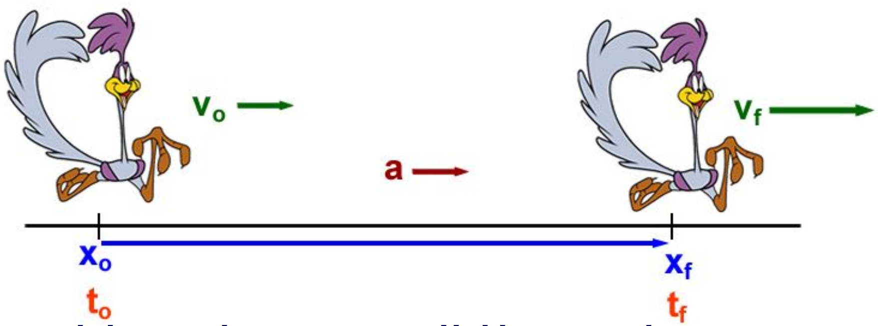
, rearrange to get

$$v^2 = v_0{}^2 + 2a\Delta x = v_0{}^2 + 2a(x - x_0)$$

- Gives velocity as a function of acceleration and displacement
- Use when you don't know and aren't asked for the time

Problem-Solving Hints

- Read the problem
- Draw a diagram
  - Choose a coordinate system, label initial and final points, indicate a positive direction for velocities and accelerations



- Label all quantities, be sure all the units are consistent
  - Convert if necessary
- Choose the appropriate kinematic equation
- Solve for the unknowns
  - You may have to solve two equations for two unknowns
- Check your results

$$v = v_0 + at$$

$$\Delta x = v_0 t + \tfrac{1}{2} a t^2$$

$$v^2 = v_0^2 + 2a\Delta x$$

Example

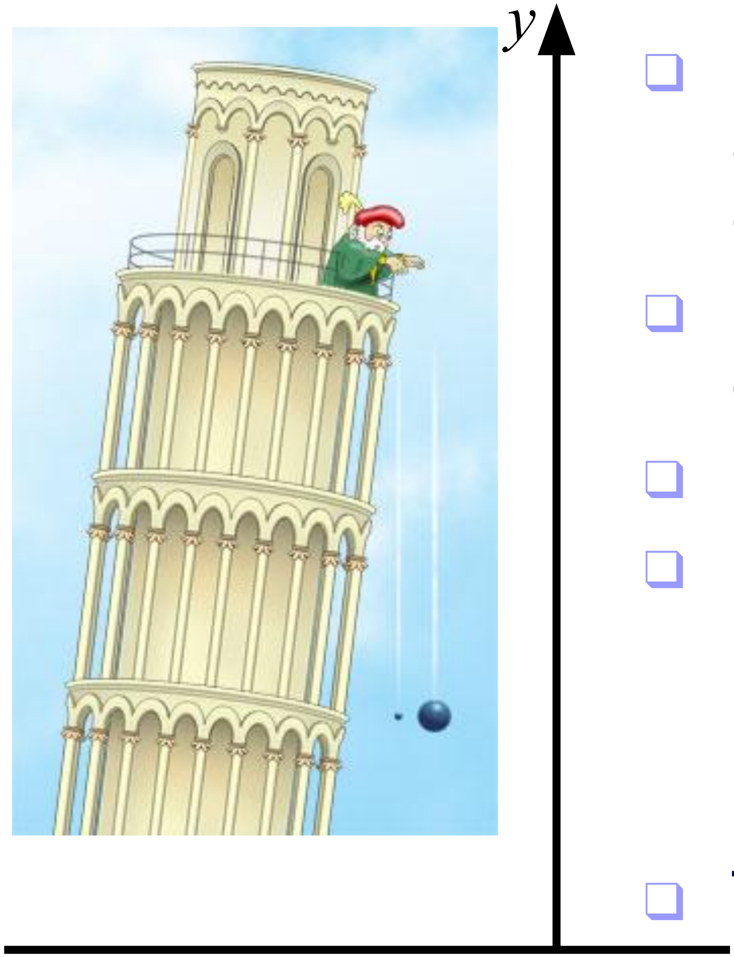
- An airplane has a lift-off speed of 30 m/s after a take-off run of 300 m, what minimum constant acceleration?

$v = v_0 + at$ $\Delta x = v_0 t + \frac{1}{2} a t^2$ $v^2 = v_0^{\ 2} + 2a\Delta x$

- What is the corresponding take-off time?

$v = v_0 + at$ $\Delta x = v_0 t + \frac{1}{2} a t^2$ $v^2 = v_0^{\ 2} + 2a\Delta x$

Free Fall Acceleration

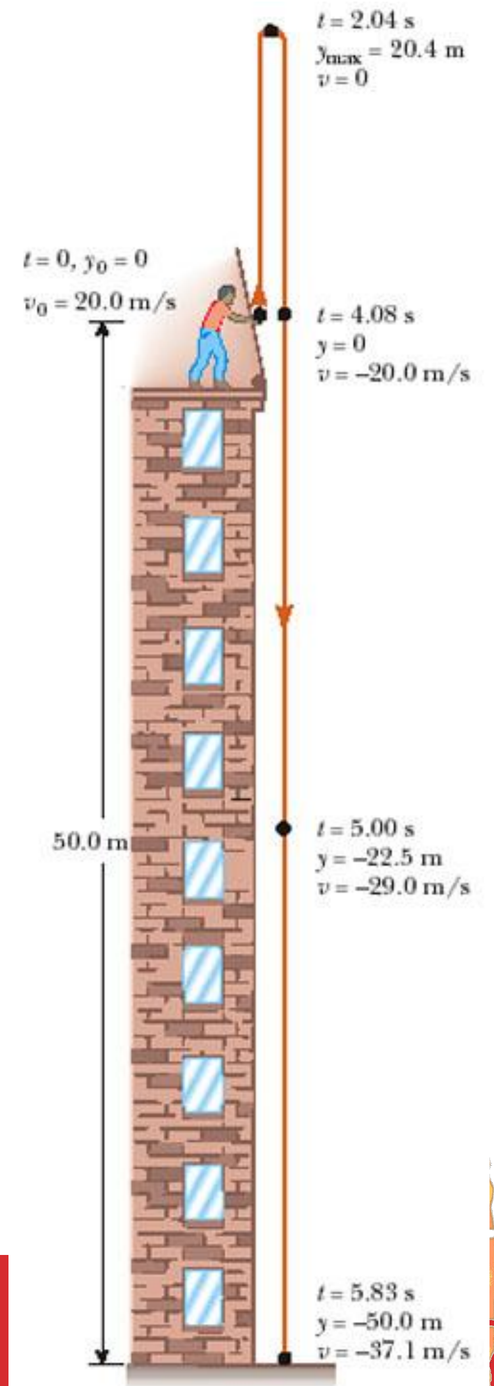


- Earth gravity provides a constant acceleration. Most important case of constant acceleration.
- Free-fall acceleration is independent of mass.
- Magnitude: $|a| = g = 9.8 \text{ m/s}^2$
- Direction: always downward, so a_g is negative if we define "up" as positive,

  $a = -g = -9.8 \text{ m/s}^2$
- Try to pick origin so that $x_i = 0$

Free Fall for Rookie

- A stone is thrown from the top of a building with an initial velocity of 20.0 m/s straight upward, at an initial height of 50.0 m above the ground. The stone just misses the edge of the roof on the its way down. Determine
- (a) the time needed for the stone to reach its maximum height.
- (b) the maximum height.
- (c) the time needed for the stone to return to the height from which it was thrown and the velocity of the stone at that instant.
- (d) the time needed for the stone to reach the ground
- (e) the velocity and position of the stone at t = 5.00s



Summary

- This is the simplest type of motion
- It lays the groundwork for more complex motion
- Kinematic variables in one dimension
  - Position $x(t)$ m L
  - Velocity $v(t)$ m/s L/T
  - Acceleration $a(t)$ m/s^2 L/T^2
  - All depend on time
  - All are vectors: magnitude and direction vector:
- Equations for motion with constant acceleration: missing quantities
  - $v = v_0 + at$
  - $x - x_0 = v_0 t + \frac{1}{2} a t^2$
  - $v^2 = v_0^2 + 2a(x - x_0)$
  - $x - x_0 = \frac{1}{2}(v + v_0)t$
  - $x - x_0 = vt - \frac{1}{2} a t^2$