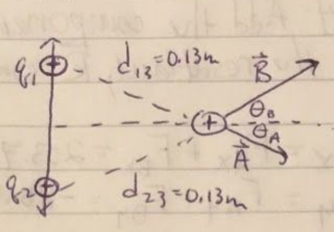
Superposition Principle Example
(Two-Dimensional)

A $16\mu C$ charge q_1 and a $28\mu C$ charge q_2 are 0.05 m from the x-axis. A $26\mu C$ charge q_3 is 0.12 m from the y-axis. The distances d_{13} and d_{23} are 0.13 m. Find the magnitude and direction of the resulting vector $\vec{R}$.

Step 1: Identify vectors



Step 2: Calculate magnitudes of $\vec{A}$ and $\vec{B}$ using $F_e = k\frac{(q_1)(q_2)}{d^2}$

Ⓐ $F_e = k\frac{q_1 q_3}{d^2} = (8.99\times10^9)\frac{(16\times10^{-6}c)(26\times10^{-6}c)}{(0.13\,m)^2} = 221.3\text{ N}$

This is the force between q_1 and q_3

Ⓑ $F_e = k\frac{q_2 q_3}{d^2} = (8.99\times10^9)\frac{(28\times10^{-6}c)(26\times10^{-6}c)}{(0.13\,m)^2} = 387.3\text{ N}$

This is the force between q_2 and q_3

Step 3: Calculate the components of $\vec{A}$ and $\vec{B}$

Ⓐ Our angle will be $\sin\theta_A = \frac{0.05m}{0.13m}$ ← distance from x-axis / ← distance between q_1 and q_3

$$\theta_A = \sin^{-1}\left(\frac{0.05m}{0.13m}\right) = 23°$$

To find components, use the shortcuts $F_{Ax} = A\cos\theta$ and $F_{Ay} = -A\sin\theta$

$F_{Ax} = (221.3\text{ N})\cos(23°)$ $\quad$ $F_{Ay} = (-221.3\text{N})\sin(23°)$

$F_{Ax} = 203.7\text{ N}$ $\quad$ $F_{Ay} = -86.5\text{ N}$

* The 221.3 N came from Step 2 *

(B) Our angle will be $\sin\theta_B = \frac{0.12\,m}{0.13\,m}$ ← distance from y-axis / ← distance between q_2 and q_3

$$\theta_B = \sin^{-1}\left(\frac{0.12m}{0.13m}\right) = 23^\circ$$

To find components, use the shortcuts $F_{Bx} = B\cos\theta$ and $F_{By} = +B\sin\theta$

$F_{Bx} = (387.3N)\cos(23^\circ)$ $\qquad$ $F_{By} = (+387.3N)\sin(23^\circ)$

$F_{Bx} = 356.5\,N$ $\qquad$ $F_{By} = +151.3\,N$

* The 387.3N came from Step 2 *

Step 4: Add the components of $\vec{A}$ and $\vec{B}$ together to get the resultant, $\vec{R}$ components

$$R_x = F_{Ax} + F_{Bx} = 203.7N + 356.5N = 560.2N$$
$$R_y = F_{Ay} + F_{By} = -86.5N + 151.3N = 64.8N$$

Step 5: Use the Pythagorean Theorem to find the magnitude of R.

$$R_x^2 + R_y^2 = R^2$$
$$(560.2N)^2 + (64.8N)^2 = R^2$$
$$313824 + 4199 = R^2$$
$$318023 = R^2$$
$$563.9N = R$$

Step 6: Use tangent to find the direction of $\vec{R}$.

$$\tan\theta = \frac{R_y}{R_x} = \frac{64.8N}{560.2N}$$

$$\theta = \tan^{-1}\left(\frac{64.8N}{560.2N}\right) = 6.6^\circ$$

Solution: $\vec{R}$ has a magnitude of 563.9N and is 6.6° North of East